Phase noise of two wavelength coherent imaging system as function of spatial frequency content

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Two wavelength coherent imaging is a technique that offers advantages over conventional coherent imaging. A significant advantage is the ability to derive range information from the phase contrast image at a known difference frequency. Phase noise detracts from the accuracy at which this range information can be extracted. We therefore describe a method for developing a relation of phase noise relative to the correlation of the image planes corresponding to each wavelength. A previously derived equation is modified and extended beyond the general case, which allows for the calculation of a correlation between each image field at various spatial frequencies. This correlation coefficient can be used to generate a probability distribution function which represents the overall phase noise of the system relative to the spatial frequency content. In general, this spatial frequency content is based on the tilt angle of the target. We discuss both a computer based model of the analytic equation, as well as an experimental spatial heterodyne verification of said model. In the future, the model will be adapted for more complex scenes.

1: Introduction & Theory

Digital holography, also sometimes called coherent imaging, is a powerful technique for imaging three-dimensional (3D) objects. Conventional coherent imaging allows for reconstruction of an object; however, multi-wavelength coherent imaging allows for the extraction of phase information. Applications are numerous, and are used in such disciplines as microscopy and laser radar (LADAR).

The advantage of using a two wavelength coherent imaging system lies in the beat frequency that is produced by two offset lasers. By tuning two lasers to a difference frequency, a beat frequency, known from here on as the synthetic wavelength, is formed. This synthetic wavelength is related to the two source wavelengths by

$$\lambda_s = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|} \quad . \tag{1}$$

Choosing a suitable synthetic wavelength allows for phase information to be extracted from a variety of objects. Although this method can be applied in various forms, in this paper it will be discussed in a spatial heterodyned setup.

Phase noise is an inherent problem in coherent imaging. The degradation of the phase contrast image by phase noise affects both image quality and the ability to extract pertinent information from the image. However, corrections can be made if the phase noise in the system can be predicted and understood. The case of a flat target with a Gaussian illumination has already been studied[1]. However, the extension of this to large angles of tilt has not yet been studied, to the knowledge of this paper's authors.

Calculating a μ value begins with generating a range profile. The range profile, defined as[2]

$$\left\langle \left|\sigma(z)\right|^2 \right\rangle = \int \left\langle \left|\sigma(x,y,z)\right|^2 \right\rangle dxdy \ , \tag{2}$$

is the time-domain reflected intensity of the target projected into *x*,*y*,*z* space. $\langle\ \rangle$ denotes an expected value.

For a flat target with Gaussian roughness, it has been shown that this range profile becomes

$$\left\langle \left|\sigma(z)\right|^2 \right\rangle_0 = \exp\left(-\pi\left(\frac{z}{a}\right)^2\right), \tag{3}$$

where z is the source to target distance and aa is a constant involving the surface roughness. This in turn can be used to generate a correlation coefficient.

However, as previously discussed, this only works for the case in which a target is flat with a known roughness. In most practical applications, a target will not be completely flat. By involving the weighting that the point spread function (PSF) gives as the target tilts, a correlation value can be calculated at various angles. The PSF will be compressed or stretched, depending on the tilt angle of the target.

A similar process is taken in calculating the μ value involving the PSF. First, a range profile is generated using a diffraction limited sinc function that is the PSF of our system.

$$\left\langle \left|\sigma(z)\right|^2 \right\rangle_T = \int \delta\left(z - (\tan\alpha)x\right) \left\langle \left| \operatorname{sin}c\left(\frac{Dx}{\lambda L}\right) \operatorname{sin}c\left(\frac{Dy}{\lambda L}\right) \right|^2 \right\rangle dxdy, \tag{4}$$

where α is the tilt angle, L is source to target distance, and D is the receiver aperture diameter. The delta function in Equation 4 shows that the reflection only takes place at the target, that is where $z = (\tan \alpha)x$. Since it is assumed that the target is only tilted in x, the y terms integrate out and Equation 4 can be written as

$$\left\langle \left|\sigma(z)\right|^2 \right\rangle_T = \left\langle \left| \operatorname{sin}c\left(\frac{Dz}{\lambda(\tan\alpha)L}\right) \right|^2 \right\rangle . \tag{5}$$

Beyond the flat case, the range profile becomes a convolution of the flat case's range profile and the range profile attributed to the PSF,

$$\left\langle \left|\sigma(z)\right|^2 \right\rangle = \left\langle \left|\sigma(z)^2\right| \right\rangle_0 \otimes \left\langle \left|\sigma(z)^2\right| \right\rangle_T = \exp\left(-\pi\left(\frac{z}{a}\right)^2\right) \otimes \operatorname{sin}c^2\left(\frac{Dz}{(\tan\alpha)\lambda L}\right). \tag{6}$$

A range profile can be turned into a cross-spectral correlation coefficient by[2]

$$\mu(\Delta\upsilon) = \int \left\langle \left|\sigma(z)\right|^2 \right\rangle \exp\left(\frac{-4i\pi z(\Delta\upsilon)}{c}\right) dz , \tag{7}$$

where z is source to target distance, $\Delta\nu$ is frequency separation of two sources, and c is the speed of light. Equation 7 can be recognized as a Fourier Transform kernel.

Combining equations 6 and 7, the Fourier transforms are trivial. This leads to the overall analytic equation for the correlation coefficient,

$$\mu = \exp\left(-\frac{8\pi^2 \sigma_T^2 \Delta\upsilon^2}{c^2}\right) \Lambda\left(\frac{2\Delta\upsilon(\tan\alpha)\lambda L}{Dc}\right), \tag{8}$$

where σ_T is the surface roughness of the target, and Λ denotes a triangle function.

Experimentally, the correlation coefficient, μ, gives the degree of correlation between two image fields. It is defined as

$$\mu = \frac{\overline{A_1 A_2^*}}{\sqrt{\overline{\left|A_1\right|^2}\;\overline{\left|A_2\right|^2}}} , \tag{9}$$

where A_1 and A_2 are the image plane fields corresponding to signal 1 and signal 2. Equation 9 can be used to generate a probability density function (PDF) by use of

$$p(\Delta\theta) = \left(\frac{1-\mu^2}{2\pi}\right) \frac{(1-\beta^2)^{1/2} + \beta\pi - \beta\cos^{-1}\beta}{(1-\beta^2)^{3/2}} , \tag{10}$$

which represents the overall phase noise in the system. In Equation 10 above,

$$\beta = \mu\cos(\Delta\theta). \tag{11}$$

The value calculated from Equation 9 should match the analytic equation derived in Equation 8. The subsequent PDFs generated should match the overall phase noise spectrum.

2: Modeling a system

A two wavelength propagation simulation was designed in order to compare results to the analytically derived equation. This simulation used two sources with a frequency separation of 50GHz, which was chosen to match the eventual experiment. All processing was done assuming a flat target with Gaussian illumination. The model agrees with the relationship between the results of Equations 8 and 9 at any desired wavelength or target tilt. It also allowed us to match these results to the phase noise PDF of the target. Figure 1 below shows a comparison of the PDF generated by combining Equations 8 and 10, as compared to a histogram of the phase noise for the simulated system. This is shown at an arbitrary angle away from zero degrees of tilt.

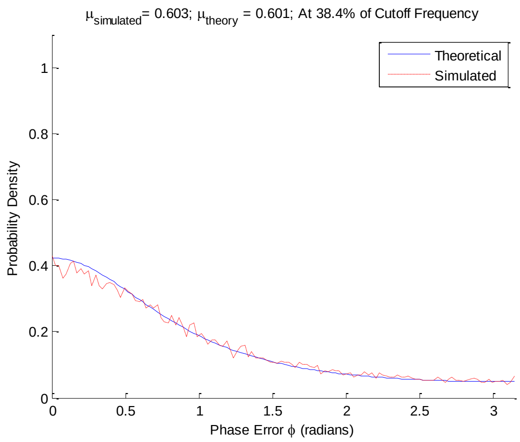

Figure 1: Verification of Analytic Equation Representing Phase Noise Spectrum

The percentage of cutoff frequency in Figure 1 is related to the tilt of the target by

$$\frac{\tan(\theta)\dfrac{\lambda z}{D}}{\lambda_S}, \tag{12}$$

where θ is target tilt angle in radians, D is receiver aperture diameter, z is source to target distance, λ is the optical wavelength, and λ_s is the synthetic wavelength.

Figure 1 above shows that both $\mu_{simulated}$ and μ_{theory} are close in value. The dashed curve is the actual phase noise spectrum of the synthetic target, which closely matches the PDF generated using μ_{theory}. Thus, the analytic equation seems to be able to predict for a target what its phase noise spectrum looks like.

3: Experiment

A spatial heterodyned experiment was designed in order to match the results of the model, and therefore confirm that the analytically derived equation was correct. A schematic of the experiment, using a dimpled metal planar target is shown below in Figure 2.

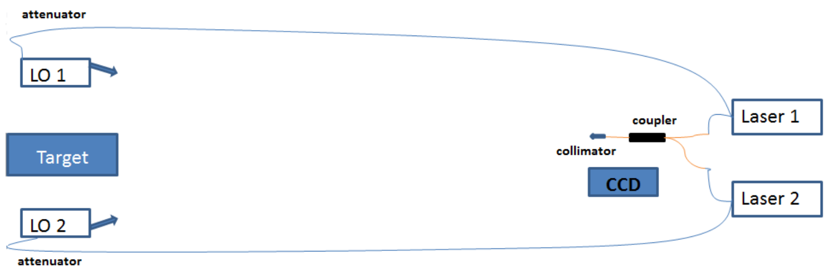

Figure 2: Schematic of Experimental Setup

Both lasers simultaneously illuminate the target. It has been shown that this has no significant effect on the cross-talk or noise of the system[3]. These lasers, which are separated by 50GHz, give a synthetic wavelength of 3mm. The roughness is on the order of 100x the optical wavelength of 1.545µm, so 3mm is sufficiently large so that the additional noise attributed to roughness is small compared to that induced by the target tilt.

The target was placed on a rotational stage with degree gradations. Data was taken in steps from zero to twenty-five degrees (Nyquist, solved from Eq. 12) in one degree steps. µ values were calculated at each of these angles and compared to the theoretical value. Figure 3 shows the changing of µ values across the entire angular spectrum.

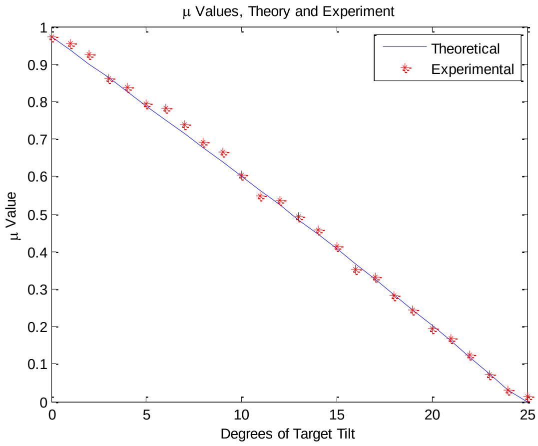

Figure 3: Correlation Coefficient, Theory and Measured

Note than in Figure 3 above, all values are within 5% of the theoretical value. This is an acceptable experimental error. In order to verify that the experiment matched the model, the same measurement as that in Figure 1 was taken in the lab. The results of this are shown below in Figure 4.

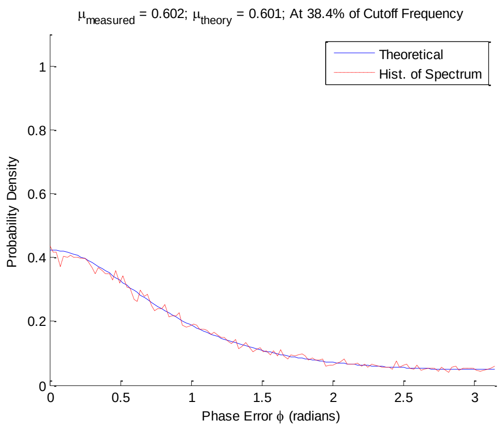

Figure 4: Experimental Verification

Note that the $\mu_{simulated}$ in Figure 1 is almost identical to the $\mu_{measured}$ in Figure 4. The experimental results line up well with the computerized model. Based upon this, Equation 8 is a valid representation of the correlation coefficient for a target in a spatially heterodyned setup.

4: Conclusions

In summary, we have discussed a correlation coefficient that allows one to accurately predict the phase noise of a target based on the overall roughness and target tilt. We have shown that the original equation for the correlation coefficient can be extended beyond a flat target. While a planar target was still used in the simulation and experiment, this equation could be extended for complex targets. There are many future applications of this technology, especially in holography.

5: References

[1] J.W. Goodman, *Speckle Phenomena in Optics (Theory and Applications)*, Roberts & Company, Englewood, Colorado, 63-68 (2007)
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[3] J. Kühn, T. Colomb, F. Montfort, *et al.*, "Real-time dual-wavelength digital holographic microscopy with a single hologram acquisition," Opt. Express **15**, 7231-7242 (2007).